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ABSTRACT: Generating an optimized pricing plan includes accessing a hierarchy comprising a sequence of levels, where each level includes an objective function and a set of constraints associated with the objective function. A mathematical programming model representing a pricing plan problem is determined for an item group comprising items. The following is repeated for each level of the sequence of levels: selecting a level comprising an objective function and a set of constraints associated with the objective function, determining an optimized boundary for the objective function, and adding a constraint generated from the optimized boundary to the set of constraints of a next level. An objective function of a last level of the sequence is optimized subject to the set of constraints associated with the objective function to yield an optimized result. An optimized pricing plan is generated in accordance with the optimized result.

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Abstract Paragraph - ABTX (1): Generating an optimized pricing plan includes accessing a hierarchy comprising a sequence of levels, where each level includes an objective function and a set of constraints associated with the objective function. A mathematical programming model representing a pricing plan problem is determined for an item group comprising items. The following is repeated for each level of the sequence of levels: selecting a level comprising an objective function and a set of constraints associated with the objective function, determining an optimized boundary for the objective function, and adding a constraint generated from the optimized boundary to the set of constraints of a next level. An objective function of a last level of the sequence is optimized subject to the set of constraints associated with the objective function to yield an optimized result. An optimized pricing plan is generated in accordance with the optimized result.

Summary of Invention Paragraph - BSTX (2): [0001] A business generates a pricing plan to determine the prices of items offered by the business such as products, goods, or services. Pricing decisions may be important since these decisions may have an impact on customer demand, profitability, and business operations. Determining prices may be complex if a collection of interdependent items needs to be priced across different channels, for example, different stores having different competitors. The pricing plan may be required to reflect the competitive situation, while at the same time satisfying business objectives such as profitability as well as targets for revenues, market share, or company price image. In addition, there may be business constraints

that need to be met. Consequently, determining an optimized pricing plan has posed challenges for businesses.

Summary of Invention Paragraph - BSTX (4): [0002] In accordance with the present invention, disadvantages and problems associated with previous techniques for determining pricing plans may be reduced or eliminated.

Summary of Invention Paragraph - BSTX (5): [0003] According to one embodiment of the present invention, generating an optimized pricing plan includes accessing a hierarchy comprising a sequence of levels, where each level includes an objective function and a set of constraints associated with the objective function. A mathematical programming model representing a pricing plan problem is determined for an item group comprising items, the mathematical programming model comprising a set of initial constraints. The following is repeated for each level of the sequence of levels: selecting a level comprising an objective function and a set of constraints associated with the objective function, adding the set of constraints associated with the objective function to the set of initial constraints, optimizing the mathematical programming model to yield an optimized boundary for the objective function, and adding a constraint generated from the optimized boundary to the set of constraints of a next level. An objective function of a last level of the sequence is optimized subject to the set of constraints associated with the objective function to yield an optimized result. The set of constraints includes a constraint generated from the optimized boundary of a previous level. An optimized pricing plan is generated in accordance with the optimized result. The optimized pricing plan associates a price with each item of the item group.

Summary of Invention Paragraph - BSTX (6): [0004] Certain embodiments of the invention may provide one or more technical advantages. A technical advantage of one embodiment may be that a pricing plan problem may be optimized according to a hierarchy of objectives, for example, maximizing profits while maintaining consistent pricing and a given sales volume. Optimizing a pricing plan problem according to a hierarchy of objectives may provide a pricing plan that better fits a company's objectives. Another technical advantage of one embodiment may be that pre-processing may be performed to set up a pricing plan problem. Pre-processing may include, for example, identifying and eliminating inconsistent constraints, dividing a category of items into more manageable item groups, and determining goals for individual time intervals, item groups, or locations. Another technical advantage of one embodiment may be that post-processing may be performed to conform optimized results to business constraints that might not have been taken into account during optimization. Post-processing may include, for example, rounding prices in accordance with rounding rules. Since price changes are known at this point, post-processing may include evaluating the cost of changing prices and adjusting prices according to the cost evaluation and prioritizing price changes according to priority rules.

Brief Description of Drawings Paragraph - DRTX (5): [0009] FIG. 3 is a flowchart illustrating an example method for performing pre-processing to set up a pricing plan problem for a mathematical programming model;

Detail Description Paragraph - DETX (2): [0013] FIG. 1 illustrates an example system 10 that generates an optimized pricing plan for a category of items. A category may include one or more groups of items offered by a business such as parts, products, or services. Items may be related to each other by, for example, a cross price demand sensitivity or other constraint. A pricing plan lists prices for the items of a category at successive time intervals across a time period. A pricing plan may be subject to a hierarchy of objectives, for example, maximizing profits while maintaining consistent pricing and a given sales volume. A business may use system 10 to determine an optimized pricing plan for items offered by the business.

Detail Description Paragraph - DETX (5): [0016] Server system 24 manages applications that generate an optimized pricing plan, such as an optimizer 28, workflows 32, and a demand analytics module 36. Optimizer 28 generates a mathematical programming model that represents a pricing plan problem, and optimizes the mathematical programming model in order to determine an optimized pricing plan. Optimizer 28 may include a pre-processing module 40, a mathematical programming module 42, and a post-processing module 44. Pre-processing module 40 performs pre-processing to set up the pricing plan problem. For example, pre-processing module 40 may identify and eliminate inconsistent constraints, divide a category into more manageable item groups, and determine goals for individual time intervals, item groups, or locations. Pre-processing module 40, however, may perform any function suitable for setting up the pricing plan problem.

Detail Description Paragraph - DETX (6): [0017] Mathematical programming module 42 generates a mathematical programming model having objectives and constraints formulated by mathematical equations and inequalities. According to one example, the mathematical programming model may include a non-linear relation, and may be solved using non-linear programming (NLP) techniques such as, for example, a reduced-gradient technique or a projected augmented Lagrangian technique. The mathematical programming model may include item costs, item demand forecasts, item allowed price ranges, a price-demand sensitivity model, a cross-price demand sensitivity model, price link constraints, price band constraints, a base price, competitor prices for a set of image items, an item inventory, a price image model, as well as constraints that model profit, revenues, image index, and other features. Mathematical programming module 42 may include a modeling language 46, a solver 48, and an application program interface (API) 50. Modeling language 46 may include, for example, A Mathematical Programming Language (AMPL) developed at Bell Laboratories, General Algebraic Modeling System (GAMS) by GAMS DEVELOPMENT CORPORATION, Advanced Interactive Mathematical Modeling Software (AIMMS) by PARAGON DECISION TECHNOLOGY B.V., or any other language suitable for modeling a pricing plan problem. Solver 48 optimizes the mathematical programming model to yield optimized results. Solver 48 may include, for example, a nonlinear programming solver such as MINOS by STANFORD BUSINESS SOFTWARE, INC., CONOPT by ARKI CONSULTING AND DEVELOPMENT A/S, or any other mathematical programming solver operable to optimize a pricing plan problem. Application program interface 50 may provide a link between optimizer 28 and server system 24.

Detail Description Paragraph - DETX (7): [0018] Post-processing module 44 performs post-

processing to conform optimized results generated by solver 48 to business constraints that might not have been taken into account during optimization. For example, post-processing module 44 may round prices in accordance with rounding rules, evaluate the cost of changing prices and adjust prices according to the cost evaluation, and prioritize price changes according to priority rules. Post-processing modules 44, however, may perform any function suitable for conforming the optimized result to business constraints.

Detail Description Paragraph - DETX (8): [0019] Workflows 32 supply information for formulating the pricing plan problem. Workflows 32 may include, for example, demand planning workflows 54, replenishment planning workflows 56, and merchandise planning workflows 58. Demand planning workflows 54 may be used to forecast a demand by, for example, determining a demand change in response to a price change. Replenishment planning workflows 56 may be used to ensure that inventories have an adequate supply of items in order to satisfy an optimized pricing plan. Merchandise planning workflows 58 may describe pricing goals for the items. For example, a pricing goal may require low prices for dairy items and higher prices for cleaning items.

Detail Description Paragraph - DETX (9): [0020] Demand analytics module 36 calculates price elasticity, which describes how a price change affects a demand. The price elasticity of a demand may be defined as the ratio of a percentage increase in demand over a percentage decrease in price. According to this definition, price elasticity is usually non-negative due to the inverse relationship between demand and price. Cross price elasticity measures how a price change of one item affects a demand of another item. Cross price elasticity of a demand may be defined as a percentage increase in demand of an item resulting from a percentage increase in price of another item.

Detail Description Paragraph - DETX (10): [0021] Cross price elasticity may be positive, negative, or zero. A positive cross price elasticity implies that the demand of an item increases if the price of another item decreases (due to, for example, cross-selling items together), whereas a negative cross price elasticity implies that the demand of an item decreases if the price of another item decreases (due to, for example, a substitution effect). A zero cross price elasticity implies that the demand of an item is not affected by the price of another item. Demand models may be used to calculate price elasticity.

Detail Description Paragraph - DETX (11): [0022] Demand analytics module 36 evaluates and selects appropriate demand models and calculates price elasticity using the selected demand models. According to one embodiment, demand analytics module 36 accesses a number of demand models and demand data describing an item group. The demand models are evaluated in accordance with the demand data, and a demand model is selected in response to the evaluation. Demand analytic information such as a demand forecast and price elasticity may be calculated using the selected demand model.

Detail Description Paragraph - DETX (12): [0023] A database 60 stores data that may be used by server system 24. Database 60 may be local to or remote from server system 24, and may be

coupled to server system 24 using one or more local area networks (LANs), metropolitan area networks (MANs), wide area networks (WANs), a global computer network such as the Internet, or any other appropriate wire line, wireless, or other links. Database 60 may include, for example, constants 64, variables 66, objectives 68, rules 69, constraints 70, demand data 71, and demand models 72. Constants 64 and variables 66 are used to formulate a set of initial constraints for a mathematical programming model representing a pricing plan problem. Constants 64 may include, for example, the following:

Detail Description Paragraph - DETX (15): [0026] Constraints 70 restrict the optimization of objective functions 68. Constraints 70 may, for example, restrict prices in response to a cost to produce an item, a manufacturer's suggested retail price, a competitor's price, or a maximum price change restriction. Constraints 70 may link related items such as different quantities of the same item or different brands of the same item. Such constraints 70 may include, for example, price-link constraints or price band constraints such as item-price inequalities. Constraints 70 may restrict prices with respect to a demand, for example, demand-price relations or cross-price elasticities. Constraints 70 governing price changes such as a minimum time between subsequent price changes of an item, a maximum number of simultaneous price changes of all items within one optimization period, or a price-implementation cost constraint may be included. Rules 69 for controlling inventory may be used to avoid stock-outs or to accommodate lead-times. Rules 69 may be directional, such that the price of a first item may affect the price of a second item, but the price of the second item does not affect the price of the first item. Rules 69 may be received from any suitable source, such as from a user of client system 20 or from price and demand analytics module 36. The following is an example of an objective function 68 and constraints 70, where profit is maximized subject to constraints.

Detail Description Paragraph - DETX (22): [0030] Demand data 71 includes data that may be used to determine a price elasticity. Demand data 71 may include, for example, sales history, price history, competitor's prices, inventory availability, and other information that may be used to determine the relationship between price and demand. Demand models 72 may be applied to demand data 71 to determine price elasticity. Demand models 72 may include, for example, constant elasticity static models and models with coefficients varying according to functions such as polynomial or log functions.

Detail Description Paragraph - DETX (24): [0032] The method begins at step 98, where optimizer 28 receives input data. Input data may include a demand forecast, competitor prices, price demand sensitivity, cross-price sensitivity, and other price-related information. At step 100, pre-processing module 40 performs pre-processing in order to set up a pricing plan problem. Pre-processing is described in more detail with reference to FIG. 3. Mathematical programming module 42 generates a mathematical programming model of the pricing plan problem at step 102. The mathematical programming model may be expressed in modeling language 46.

Detail Description Paragraph - DETX (26): [0034] The optimization method may be performed multiple times from different starting points in order to generate multiple locally optimal pricing plans. The pricing plans may be evaluated to determine a globally optimal pricing plan.

Detail Description Paragraph - DETX (30): [0038] FIG. 4 is a flowchart illustrating an example method for optimizing a mathematical programming model subject to a hierarchy of objectives and constraints. The method begins at step 150, where a hierarchy comprising levels of objectives and constraints is created. A hierarchy may include, for example, a primary objective of maximizing revenue, a secondary objective of maximizing sales, and a tertiary objective of maintaining consistent pricing of image items around a competitor's prices. Other objectives or constraints may be used, for example, constraining deviations from previous prices.

Detail Description Paragraph - DETX (45): [0053] FIG. 6 illustrates an example graph 210 representing the relationship between prices of items. Circles 214 represent the prices for items A through F. Propagation arrows 216 represent rules for propagating the price of one item to determine the price of another item. For example, the price of item A is propagated to determine the price of item B by multiplying the price of item A by two. Rounding arrows 218 represent rounding rules. In the illustrated example, a rounding rule states that a price must be rounded up to the next price that ends with a nine, for example, \$1.85 is rounded to \$1.89.

Detail Description Paragraph - DETX (46): [0054] According to an example rounding and propagation procedure, the price of item A is rounded. After the price of item A is rounded, the price is propagated to determine the price of item B. The price of item B is rounded, and then propagated to determine the price of item C, and so on. Once a price is determined, it is fixed to ensure consistent pricing. For example, once the price of item D is determined to be \$5.69, it is not changed in accordance with a propagation from item E. Prices that are not linked with other prices are rounded independently of the rounding and propagation of the prices. For example, the price of item F is rounded independently of the rounding and propagation of other prices.

Detail Description Paragraph - DETX (47): [0055] Returning to FIG. 5, the performance of the rounded and propagated prices is reevaluated at step 184. The reevaluation may be used to determine a change such as a decrease in performance resulting from applying the rounding and propagation procedure. Only after applying the rules 69 is it known which prices the overall method recommends to change. The cost of changing the prices is evaluated at step 186. The cost may measure, for example, an operational cost of implementing price changes. Prices are adjusted in accordance with the cost evaluation at step 188. For example, small price changes may be set back to an original price in order to avoid perturbation through small changes and an operational cost resulting from implementing the price changes. Post-processing module 44 enforces that the price changes meet a minimum required relative or absolute price change as compared to the original item price. Post-processing module 44 can either reset prices to the current prices or enforce the minimum required price change. Also, the improvement of performance resulting from a price change (for example, margin improvement over the original price) may be calculated in order to determine whether the price change justifies the cost incurred for implementing the price change. Post-processing module 44 determines whether the number of price changes exceed a predetermined maximum number of price changes at step 190. If the number of price changes exceeds a maximum number of price changes, post-processing module 44 may proceed to steps 192, 194, or 196 in order to determine which prices to change. Steps 192, 194, and 196 may be used to determine which prices are to be changed, or may be used to prioritize prices such that

higher priority prices are changed during one time interval, and lower priority prices are changed during a next time interval. Steps 192, 194, and 196 may be performed in any suitable order. Any other suitable procedure for prioritizing price changes may be used at steps 192, 194, and 196.

Detail Description Paragraph - DETX (48): [0056] For example, at step 192, the prices of items with the larger price changes receive the highest priority and are changed. Larger price changes may have a greater impact and are less likely to change again in comparison with smaller price changes. At step 194, prices of image items used to track competitors are changed because image items may be regarded as more important. At step 196, the prices of items with the more stable prices are changed. Items with the more stable prices may include for example, items that have not had a price change during a long time period or those items that have had the fewest price changes in a given time period. Changing the more stable prices may reduce the frequency of price changes, which may provide for more consistent pricing.

Detail Description Paragraph - DETX (49): [0057] If the number of price changes does not exceed a maximum number of price changes at step 190, the method proceeds directly to step 198. Performance under the revised pricing plan is re-evaluated at step 198. The performance may be re-evaluated in order to allow for comparison with the optimization results. The results are reported at step 200, and the method ends.

Detail Description Paragraph - DETX (50): [0058] Certain embodiments of the invention may provide one or more technical advantages. A technical advantage of one embodiment may be that a pricing plan problem may be optimized according to a hierarchy of objectives, for example, maximizing profits while maintaining consistent pricing and a given sales volume. Optimizing a pricing plan problem according to a hierarchy of objectives may provide a pricing plan that better fits a company's objectives. Another technical advantage of one embodiment may be that pre-processing may be performed to set up a pricing plan problem. Pre-processing may include, for example, identifying and eliminating inconsistent constraints, dividing a category of items into more manageable item groups, and determining goals for individual time intervals, item groups, or locations. Another technical advantage of one embodiment may be that post-processing may be performed to conform optimized results to business constraints that might not have been taken into account during optimization. Post-processing may include, for example, rounding prices in accordance with rounding rules, evaluating the cost of changing prices and adjusting prices according to the cost evaluation, and prioritizing price changes according to priority rules.

Detail Description Table CWU - DETL (1): 1 G Group of items to be optimized together, for example, a group of items that are directly or indirectly related by constraints or by a cross-price demand sensitivity; I Sub-group of certain items used to track a competitor's prices, referred to as "image items"; l.sub.ij Price link constraint between items i and j constraining their prices using an equality relation that may additionally include an additive term l.sup.a.sub.ij; b.sub.ij Price band constraint between items i and j constraining the range of one item with respect to the other item using a relation that may additionally include an additive term b.sup.a.sub.ij; e.sub.ij Elasticity and cross-elasticity lift model for the sensitivity of demand of item j based on price of item i and base price of item i; e.sub.ij being a function that computes both the own-price and

cross-price demand lifts;  $q_{sub.i}$  Base price of item  $i$ , on which the demand forecast for item  $i$  is based;  $c_{sub.i}$  Cost of item  $i$ ;  $p_{sub.i}$ , [overscore ( $p_{sub.i}$ )] Lower bound and upper bound for price of item  $i$ ;  $o_{sub.i}$  Competitor's price of item  $i$ ;  $f_{sub.i}$  Demand forecast for an optimization period based on base price  $q_{sub.i}$  of item  $i$ ; and  $u_{sub.i}$  Available inventory of item  $i$ .

Detail Description Table CWU - DETL (2): 2  $p_{sub.1}$  Price of item  $i$ ; 1  $d_{ij} = i G e_{ij} (p_i, q_i)$   $f_i$  Forecasted demand for item  $i$  in an optimization period, given price  $p_{sub.i}$ ;  $m_{sub.j} = d_{sub.j} \cdot \text{multidot.} (p_{sub.i} - c_{sub.i})$  Forecasted profit from selling item  $j$  according to forecasted demand; 2  $I_j = i I p i o i I$  Price image index over the image items used to track certain competitor's prices; 3  $M = i G m i$  Total profit from selling all items; 4  $R = i G (p i d i)$  Total revenues from selling all items; and  $P = M / (R + .\text{epsilon.})$  Total profit percentage for which a small fractional term  $.\text{epsilon.}$  is used to avoid division-by-zero on zero-revenues.

Claims Text - CLTX (2): 1. A method for generating an optimized pricing plan, comprising: accessing a hierarchy comprising a sequence of levels, each level comprising an objective function and a set of constraints associated with the objective function; determining a mathematical programming model representing a pricing plan problem for an item group comprising a plurality of items, the mathematical programming model comprising a set of initial constraints; repeating for each level of the sequence of levels: selecting a level comprising an objective function and a set of constraints associated with the objective function; adding the set of constraints associated with the objective function to the set of initial constraints; optimizing the mathematical programming model to yield an optimized boundary for the objective function; and adding a constraint generated from the optimized boundary to the set of constraints of a next level; optimizing an objective function of a last level of the sequence subject to the set of constraints associated with the objective function to yield an optimized result, the set of constraints comprising a constraint generated from the optimized boundary of a previous level; and generating an optimized pricing plan in accordance with the optimized result, the optimized pricing plan associating a price with each item of the item group.

Claims Text - CLTX (8): 7. The method of claim 1, further comprising: accessing a rounding rule for rounding a price of an item for each item of the item group; accessing a plurality of propagation rules, each propagation rule for determining a price of a first item according to a price of a second item, each propagation rule associated with an item of the item group; and repeating the following for each item of the item group: applying the rounding rule to the item of the item group; and applying a propagation rule for determining the price of the item according to a price of another item, if the item is associated with a propagation rule.

Claims Text - CLTX (9): 8. The method of claim 1, further comprising: determining a price change for each item in accordance with the optimized pricing plan; assessing a cost associated with each price change; and adjusting a price change in response to a cost associated with the price change.

Claims Text - CLTX (10): 9. The method of claim 1, further comprising: determining a price



change for each item in accordance with the optimized pricing plan; and prioritizing the price changes by associating a greater price change with a higher priority and associating a smaller price change with a lower priority.

Claims Text - CLTX (12): 11. The method of claim 1, further comprising: accessing a plurality of demand models; accessing demand data describing the item group; evaluating the demand models in accordance with the demand data; selecting a demand model of the evaluated demand models in response to the evaluation; calculating a price elasticity using the selected demand model; and generating a constraint of the hierarchy in accordance with the price elasticity.

Claims Text - CLTX (13): 12. A system for generating an optimized pricing plan, comprising: a database operable to store a hierarchy comprising a sequence of levels, each level comprising an objective function, and a set of constraints associated with the objective function; and a server system coupled to the database and operable to: determine a mathematical programming model representing a pricing plan problem for an item group comprising a plurality of items, the mathematical programming model comprising a set of initial constraints; repeat for each level of the sequence of levels: selecting a level comprising an objective function and a set of constraints associated with the objective function; adding the set of constraints associated with the objective function to the set of initial constraints; optimizing the mathematical programming model to yield an optimized boundary for the objective function; and adding a constraint generated from the optimized boundary to the set of constraints of a next level; optimize an objective function of a last level of the sequence subject to the set of constraints associated with the objective function to yield an optimized result, the set of constraints comprising a constraint generated from the optimized boundary of a previous level; and generate an optimized pricing plan in accordance with the optimized result, the optimized pricing plan associating a price with each item of the item group.

Claims Text - CLTX (19): 18. The system of claim 12, wherein the server system is operable to: access a rounding rule for rounding a price of an item for each item of the item group; access a plurality of propagation rules, each propagation rule for determining a price of a first item according to a price of a second item, each propagation rule associated with an item of the item group; and repeat the following for each item of the item group: applying the rounding rule to the item of the item group; and applying a propagation rule for determining the price of the item according to a price of another item, if the item is associated with a propagation rule.

Claims Text - CLTX (20): 19. The system of claim 12, wherein the server system is operable to: determine a price change for each item in accordance with the optimized pricing plan; assess a cost associated with each price change; and adjust a price change in response to a cost associated with the price change.

Claims Text - CLTX (21): 20. The system of claim 12, wherein the server system is operable to: determine a price change for each item in accordance with the optimized pricing plan; and prioritize the price changes by associating a greater price change with a higher priority and associating a smaller price change with a lower priority.

Claims Text - CLTX (23): 22. The system of claim 12, wherein the server system is operable to: access a plurality of demand models; access demand data describing the item group; evaluate the demand models in accordance with the demand data; select a demand model of the evaluated demand models in response to the evaluation; calculate a price elasticity using the selected demand model; and generate a constraint of the hierarchy in accordance with the price elasticity.

Claims Text - CLTX (24): 23. Software for generating an optimized pricing plan, the software encoded in media and when executed operable to: access a hierarchy comprising a sequence of levels, each level comprising an objective function and a set of constraints associated with the objective function; determine a mathematical programming model representing a pricing plan problem for an item group comprising a plurality of items, the mathematical programming model comprising a set of initial constraints; repeat for each level of the sequence of levels: selecting a level comprising an objective function and a set of constraints associated with the objective function; adding the set of constraints associated with the objective function to the set of initial constraints; optimizing the mathematical programming model to yield an optimized boundary for the objective function; and adding a constraint generated from the optimized boundary to the set of constraints of a next level; optimize an objective function of a last level of the sequence subject to the set of constraints associated with the objective function to yield an optimized result, the set of constraints comprising a constraint generated from the optimized boundary of a previous level; and generate an optimized pricing plan in accordance with the optimized result, the optimized pricing plan associating a price with each item of the item group.

Claims Text - CLTX (30): 29. The software of claim 23, operable to: access a rounding rule for rounding a price of an item for each item of the item group; access a plurality of propagation rules, each propagation rule for determining a price of a first item according to a price of a second item, each propagation rule associated with an item of the item group; and repeat the following for each item of the item group: applying the rounding rule to the item of the item group; and applying a propagation rule for determining the price of the item according to a price of another item, if the item is associated with a propagation rule.

Claims Text - CLTX (31): 30. The software of claim 23, operable to: determine a price change for each item in accordance with the optimized pricing plan; assess a cost associated with each price change; and adjust a price change in response to a cost associated with the price change.

Claims Text - CLTX (32): 31. The software of claim 23, operable to: determine a price change for each item in accordance with the optimized pricing plan; and prioritize the price changes by associating a greater price change with a higher priority and associating a smaller price change with a lower priority.

Claims Text - CLTX (34): 33. The software of claim 23, operable to: access a plurality of demand models; access demand data describing the item group; evaluate the demand models in accordance with the demand data; select a demand model of the evaluated demand models in response to the evaluation; calculate a price elasticity using the selected demand model; and generate a constraint of the hierarchy in accordance with the price elasticity.

Claims Text - CLTX (35): 34. A system for generating an optimized pricing plan, comprising: means for accessing a hierarchy comprising a sequence of levels, each level comprising an objective function and a set of constraints associated with the objective function; means for determining a mathematical programming model representing a pricing plan problem for an item group comprising a plurality of items; means for repeating for each level of the sequence of levels: selecting a level comprising an objective function and a set of constraints associated with the objective function; determining an optimized boundary for the objective function; and adding a constraint generated from the optimized boundary to the set of constraints of a next level; means for optimizing an objective function of a last level of the sequence subject to the set of constraints associated with the objective function to yield an optimized result, the set of constraints comprising a constraint generated from the optimized boundary of a previous level; and means for generating an optimized pricing plan in accordance with the optimized result, the optimized pricing plan associating a price with each item of the item group.

Claims Text - CLTX (36): 35. A method for generating an optimized pricing plan, comprising: accessing a hierarchy comprising a sequence of levels, each level comprising an objective function and a set of constraints associated with the objective function; splitting a category of items into a plurality of item groups, each item group comprising a set of items related by one or more item constraints; selecting an item group for which to generate an optimized pricing plan; determining a mathematical programming model representing a pricing plan problem for the selected item group, the mathematical programming model comprising a non-linear programming model; repeating for each level of the sequence of levels: selecting a level comprising an objective function and a set of constraints associated with the objective function; determining an optimized boundary for the objective function by expressing the objective function using one or more slack variables, and optimizing the one or more slack variables to determine the optimized boundary; and adding a constraint generated from the optimized boundary to the set of constraints of a next level; optimizing an objective function of a last level of the sequence, using a non-linear programming technique, subject to the set of constraints associated with the objective function to yield an optimized result, the set of constraints comprising a constraint generated from the optimized boundary of a previous level; accessing a rounding rule for rounding a price of an item for each item of the item group; accessing a plurality of propagation rules, each propagation rule for determining a price of a first item according to a price of a second item, each propagation rule associated with an item of the item group; repeating the following for each item of the item group: applying the rounding rule to the item of the item group, and applying a propagation rule for determining the price of the item according to a price of another item, if the item is associated with a propagation rule; and generating the optimized pricing plan in accordance with the optimized result, the optimized pricing plan associating a price with each item of the item group.